Sprint Corporation
January 22, 2002

Exhibit B

Sprint Position Statement on Statistical Testing

Notice of Proposed Rulemaking
Performance Measurements and Standards for Unbundled Network Elements and Interconnection

CC Docket No. 01-318

Generally, non-parametric tests (e.g., Permutation Tests, Fisher's Exact Test, and the Binomial Test) are the preferred methods, given the typical non-normality of performance measurement data. However, such tests can be impractical due to being computationally intensive. Thus, non-parametric tests should be employed only for "small" samples. For "large" samples, parametric tests (e.g., z-tests) should be used (along with a skewness correction when appropriate).

The definition of "small" should be made in the context of the accuracy obtained using parametric statistical testing methodologies for "large" samples. In other words, the cutoff between "small" and "large" samples should be set such that sufficient accuracy is obtained when employing "large" sample testing methodologies. Sprint proposes no minimum sample size for statistical tests. In other words, Sprint proposes that statistical tests are applicable regardless of how small the sample size may be. Even though the reliability of the statistical tests can be compromised for very small sample sizes, Sprint sees no reasonable alternative to simply testing on any sample. Others may propose accumulating transactions until a minimum sample size is met; though the administrative difficulties of this are prohibitive. Some may even propose "throwing-out" data; though this would be problematic for any high-capacity services that tend to have small transaction counts (or order volumes).

Standards should be developed for the specific statistical tests to be used, as well as the conditions for when/how to use each type of test. However, since accuracy is dependent upon the nature of the data being tested, and each ILEC/carrier-customer could have relatively unique data, such standards should be viewed as preferred methodology unless evidence is produced that demonstrates a more accurate test (given the situational nature of the data being tested). Adopting a uniform methodology, without flexibility, could result in inaccurate test results (due simply to the uniqueness of data).

See table below for appropriate statistical tests to use, as a standard, with the recognition that more accurate tests may be employed if the data so warrant.

Sample Size	**Type of Measure**	**Preferred Statistical Tests (*without* cell-level comparisons)**	**Preferred Statistical Method (cell-level comparisons)**
"small"	mean	Permutation Testing	Permutation Testing (p-value converted to a z-score)
	proportion	Fisher's Exact Test (i.e., Hypergeometric)	Standard Z, with finite population correction
	rate	Binomial Test	Standard Z, with finite population correction
"large"	mean	Modified Z, with skewness correction (ILEC variance used, rather than pooled variance)	Modified Z, with skewness correction (ILEC variance used, rather than pooled variance)
	proportion	Standard Z, with finite population correction	Standard Z, with finite population correction
	rate	Standard Z, with finite population correction	Standard Z, with finite population correction

The benefit of standards for statistical methodologies would be to lessen the burden of having vastly different methodologies state-by-state. However, uniform statistical methodology would *not* assist in comparing performance across regions. Test statistics are highly sensitive to sample size. Thus, test statistics will vary due to sample size, even if performance remains constant. In other words, 100 transactions through a process will yield a different statistical result than 1,000 transactions through the exact same process, even when all transactions are "treated equally." Test statistics do not measure performance, nor do they provide a basis by which to compare performance from one region to the next (unless sample sizes are the same across regions). Comparison of performance across regions should be made by comparing performance measurement results directly, or by comparing metrics that are not so sensitive to sample size.

Statistical testing methodologies are the preferred method for evaluating parity service. However, since parity comparisons are not always available for certain key performance measures, benchmarks may be appropriate. When used, benchmarks should be set as tolerance limits and not as performance targets. In other words, the benchmark should take into consideration the potential for random variation in the process. Doing so obviates the need for statistical testing on benchmark measures.
For benchmark proportion measures, consideration should be made for the impact that small samples can have in necessitating better-than-compliant service in order to meet the benchmark. For example, if a benchmark is set at 95%, and there are only 19 transactions, missing one transaction would yield a performance result of 94.7%. Thus, such a situation requires 100% performance. Sprint proposes that a table be designed to designate "practical benchmark" performance for small samples associated with benchmark proportion measures. In this example, the practical benchmark might allow for one miss out of the nineteen transactions.

Sprint advocates the standard confidence level of 95% (i.e., Type I error rate of 5%) for all statistical tests. In setting the Type I error rate at 5%, each statistical test has a 5% chance of causing the ILEC to pay incentives even when processes are operating at parity. Thus, Sprint proposes a forgiveness provision to account for the monetary impact to the ILEC resulting from Type I errors. The forgiveness provision, in general, would forgive payment of incentives in such as way as to mitigate the risk of Type I errors.

Since the data being tested are observational data, and not data collected through an experimental design, the accuracy of any statistical test is highly dependent upon the assumption that comparisons are being made at "like-to-like" levels. For instance, a statistical test comparing the repair intervals of ILEC retail data to a particular carrier-customer may conclude disparity due simply to the fact that the ILEC's retail customers may be mostly in rural areas, while the carrier-customer's business is concentrated in urban areas. It cannot be assumed that the unavoidable difference in repair intervals between urban and rural areas are accounted for in the definition of disaggregation of a measurement for reporting purposes. In many instances, however, the disaggregation of a measurement will indeed provide reasonably like-to-like comparisons. Further, when it is found that a reported disaggregation of a measurement does not provide for a like-to-like comparison, it may prove to be impractical and unnecessary to expand the disaggregation to account for all like-to-like comparisons (for instance, if the repair intervals are being reported by service group types, and yet a like-to-like comparison must be made at individual wire centers, it is not necessary to report each service group type by dozens of individual wire centers). Instead of accounting for all levels of like-to-like comparisons in the reporting level (disaggregated measurements), statistical comparisons can be made at like-to-like levels (called the "cell level"), and aggregated to a single test statistic at the reporting level.

When performing cell-level comparisons, Sprint proposes an aggregation technique (initially developed by Dr. Collin Mallows while working for AT&T) which will not only allow for more accurate tests at the reporting level, but which will also minimize the potential of good performance masking bad performance. See Attachment A for detailed statistical techniques.

When statistical tests are employed, Materiality Thresholds should be implemented, when appropriate, in recognition that statistical significance does not necessarily equate to business significance (see Attachment One).

When cell-level comparisons are made (i.e., statistical comparisons below the reporting level), a single aggregate test statistic, the "Truncated Z", is used for testing at the reporting level. See details in Attachment Two.

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Attachment One

Materiality Thresholds for Parity Measures

When evaluating compliance in providing carrier-customers with service that is in parity with service provided to retail customers, statistical tests can misidentify differences as significant. This weakness in the statistical tests is due simply to the fact that *statistical significance* is not necessarily synonymous with *business significance*.
The proposal ---

Certain parity measures (and/or submeasures) would have predetermined materiality thresholds. Setting these materiality thresholds would be accomplished through the same process of negotiation as are benchmark values. All statistical tests should be performed as proposed. However, when the statistical tests yield a non-compliant result, a check for materiality should be made at the submeasure level, for each carrier-customer. If the proposed materiality threshold is not crossed, despite the results of the statistical test, the result would be deemed compliant.

There are two types of materiality considerations that should be made --- one for measures typically associated with small samples and one for measures typically associated with large samples.

Small Samples for Parity Measures
For measures typically associated with small samples, the measure itself can be highly sensitive to small differences in service. Similar to the small sample adjustment used for benchmark proportion measures, small samples for parity measures (especially proportion and rate measures) can result in the need for perfect or near-perfect service in order to be deemed compliant. For example, the measure *Trouble Report Rate* is defined as the number of trouble tickets per month divided by the number of access lines the customer has. For the retail business as a whole (for a particular submeasure), there are typically 18 troubles per 36,814 access lines, for a trouble rate of 0.05%. For a particular carrier-customer with 173 access lines, a single trouble report would result in a 0.6% trouble rate. This would result in statistically significant non-compliance (z-score = -3.05). However, one trouble report for a month does not have a significant impact on the carrier-customer's ability to compete.

The proposal ---

To set the threshold of materiality for the *Trouble Report Rate* measure, the following adjustment table should be adopted:

Number of Access Lines (for a carrier-customer)	Permitted Troubles
1 to 24	1
25 to 74	2
75+	3

For the carrier-customer with 173 access lines and 1 trouble, accompanied by a statistically significant difference, a look-up in this table would indicate that more than 3 troubles would be required before a significant business impact would occur. As a note for how *not* to use this table, consider a carrier-customer with 4 troubles and better than parity service (i.e., the carrier-customer is receiving better service than the retail results). This table does not indicate that no more than 3 troubles are ever allowable. It is used only when there is a statistically significant difference identified.

Large Samples for Parity Measures

For measures typically associated with large samples, the measure is not sensitive to slight differences in service, but the resulting statistical test is. Billing measures, for example, tend to have large sample sizes. These large sample sizes make such measures sensitive to very small differences in service. For instance, suppose a retail result runs about 98.3%, based on 60,000 transactions, and a particular carrier-customer has a 97.9% result, based on 3,000 transactions. The difference in service (0.4%) is slight, but could result in a statistically significant difference. Even though the statistical test might identify a difference, it begs the question as to whether a 0.4% difference for this carrier-customer actually has any impact on that carrier-customer's ability to compete.

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Attachment Two

Statistical Calculations

Statistical functions Definitions:

$\Phi^{-1}(x)$ Inverse cumulative standard normal distribution function.

$pt(t, df)$ Cumulative distribution function of a t-statistic with df degrees of freedom.

$BN(x,n,p)$ Binomial distribution density function. The probability of observing x of n successes with a probability p of success.

$CBN(x,n,p)$ Cumulative binomial distribution function.

$$CBN(x,n,p) = P(B \le x) = \begin{cases} 0(x < 0) \\ \sum_{k=0}^{x} BN(k)(0 \le x \le n) \\ 1(x > n) \end{cases}$$

$HG(q,m,n,k)$ Hypergeometric distribution density function where q represents the number of red balls out of a sample of size k drawn from an urn containing m red balls and n black ones.

$CHG(q,m,n,k)$ Cumulative hypergeometric distribution.

$$CHG(q,m,n,k) = P(H \le q) = \begin{cases} 0(q < \max(0,k-m)) \\ \sum_{h=\max(0,k-m)}^{q} HG(h)(\max(0,k-m) \le q \le \min(k,m)) \\ 1(q > \min(k,m)) \end{cases}$$

$rank(x)$ Ranks the input variables. In case of ties, the average rank is calculated.

$choose(n,k)$ Calculates the binomial coefficients.

<u>Global variable definitions:</u>

L	=	The total number of occupied cells.[1]
j	=	An index counter indicating cell number.
n_{1j}	=	The number of ILEC transactions in cell j.
n_{2j}	=	The number of carrier-customer transactions in cell j.
n_j	=	The total number of transactions in cell j.
X_{1jk}	=	Individual ILEC transactions in cell j.
X_{2jk}	=	Individual carrier-customer transactions in cell j.
Φ^{-1}	=	Inverse cumulative standard normal distribution function.

Mean Performance Measures[2]

<u>Variable definitions:</u>

STATISTIC	***DEFINITION***	***EXPLANATION***
$\overline{X}_{1j} = \frac{1}{n_{1j}} \sum_{k=1}^{n_{1j}} X_{1jk}$	ILEC sample mean of cell j.	Add observations and divide by the number of observations.
$\overline{X}_{2j} = \frac{1}{n_{2j}} \sum_{k=1}^{n_{2j}} X_{2jk}$	Carrier-customer sample mean of cell j.	Add observations and divide by the number of observations.
$s_{1j}^2 = \frac{1}{n_{1j}-1} \sum_{k=1}^{n_{1j}} (X_{1jk} - \overline{X}_{1j})^2$	ILEC sample variance in cell j. May be NA for very small sample sizes.	Subtract each observation by its mean, square the difference, add them all up, and divide by the number of observations minus 1.
$s_{2j}^2 = \frac{1}{n_{2j}-1} \sum_{k=1}^{n_{2j}} (X_{2jk} - \overline{X}_{2j})^2$	Carrier-customer sample variance in cell j. May be NA for very small sample sizes.	Subtract each observation by its mean, square the difference, add them all up, and divide by the number of observations minus 1.

[1] If comparisons are performed at the submeasure level, L = 1 and only one cell (the submeasure) exists. If comparisons are performed at the cell level, L may exceed 1 and more than one cell may exist (see Attachment G for the list of (sub)measurements approved for comparison at the cell level).

[2] Only perform STEP 4 and STEP 5 if L > 1 (e.g., if this is a cell-level comparison, and there is more than one cell with CLEC activity, then perform STEP 4 and STEP 5).

$\gamma_{1j} = \dfrac{\dfrac{1}{n_{1j}} \sum_{k=1}^{n_{1j}} \left(X_{1jk} - \bar{X}_{1j} \right)^3}{\left[\dfrac{1}{n_{1j}} \sum_{k=1}^{n_{1j}} \left(X_{1jk} - \bar{X}_{1j} \right)^2 \right]^{3/2}}$	The ILEC sample skewness in cell j. May be NA for very small sample sizes.	Subtract each observation by its mean, cube the difference, add them all up, and divide by the number of observations. Then divide that number by the cubed square root of the population variance.
$\gamma_{2j} = \dfrac{\dfrac{1}{n_{2j}} \sum_{k=1}^{n_{2j}} \left(X_{2jk} - \bar{X}_{2j} \right)^3}{\left[\dfrac{1}{n_{2j}} \sum_{k=1}^{n_{2j}} \left(X_{2jk} - \bar{X}_{2j} \right)^2 \right]^{3/2}}$	The carrier-customer sample skewness in cell j. May be NA for very small sample sizes.	Subtract each observation by its mean, cube the difference, add them all up, and divide by the number of observations. Then divide that number by the cubed square root of the population variance.
XY_j	Combined ILEC and carrier-customer samples.	Concatenate the ILEC and carrier-customer samples into a single variable.

STEP 1: Calculate Cell Weights

$$W_j = \sqrt{\frac{n_{1j} n_{2j}}{n_j}}$$

For each cell, multiply the ILEC sample size and the carrier-customer sample size, divide by their sum, and take a square root.

If all ILEC and carrier-customer transactions within a cell have identical performance measures (e.g., service durations), set $W_j = 0$.

STEP 2: Calculate a Z-statistic for each cell

a. If $W_j = 0$, then set $Z_j = 0$.

b. If $\min(n_{1j}, n_{2j}) > 6$ and $s_{1j}^2 > 0$

$$T_j = \begin{cases} t_j + \dfrac{g}{6}\left(\dfrac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}} \right)\left(t_j^2 + \dfrac{n_{2j} - n_{1j}}{n_{1j} + 2n_{2j}} \right) & t_j \geq t_{\min j} \\ t_j + \dfrac{g}{6}\left(\dfrac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}} \right)\left(t_{\min j}^2 + \dfrac{n_{2j} - n_{1j}}{n_{1j} + 2n_{2j}} \right) & \text{otherwise} \end{cases},$$

where

$$t_j = \frac{\bar{X}_{1j} - \bar{X}_{2j}}{s_{1j}\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}},$$

$$t_{\min j} = \frac{-3\sqrt{n_{1j}n_{2j}n_j}}{g(n_{1j} + 2n_{2j})}$$

and g is the median value of all values of γ_{1j} over all cells within the submeasure (reporting level) such that

i) $\gamma_{1j} > 0$

ii) $n_{1j} > 6$, and

iii) $n_{1j} > n_{3q}$, where n_{3q} is the 3 quartile of all n_{1j}.in cells where (i) and (ii) are true.

If no cells within a submeasure exist that satisfy conditions (i) - (iii), then set g = 0.

Calculate the p-value from the T_j statistic with $n_{1j} - 1$ degrees of freedom using $P_j = pt(T_j, n_{1j} - 1)$.
Calculate the z-score Z_j from this p-value as $Z_j = \Phi^{-1}(P_j)$.

c. If [$\min(n_{1j}, n_{2j}) \le 6$ OR $s_{1j}^2 = 0$] AND $W_j > 0$ (from part 1):

1) Calculate the number of possible permutations
Nperms $= choose(n_j, n_{1j})$

2) If $n_{1j} = n_{2j} = 1$, then $Z_j = \begin{cases} 0.6744898 & X_{1j} > X_{2j} \\ 0 & X_{1j} = X_{2j} \\ \text{-}0.6744898 & X_{1j} < X_{2j} \end{cases}$

3) If only $n_{1j} = 1$ then let R_0 equal the rank of the ILEC observation in the combined sample XY_j. Calculate $Z_j = \Phi^{-1}\left(\frac{R_0 - 0.5}{n_j}\right)$.

4) If only $n_{2j} = 1$ then let R_0 equal the rank of the carrier-customer observation in the combined sample XY_j. Calculate $Z_j = -\Phi^{-1}\left(\frac{R_0 - 0.5}{n_j}\right)$.

5) If $\min(n_{1j}, n_{2j}) \ge 2$ and $Nperms \le 1000$ then

   i) Generate all possible permutations of sizes n_{1j} and n_{2j} from the combined sample XY_j.

ii) For each permuted sample, calculate the sum of sample of size n_{1j}.

iii) Let R_0 equal the rank of the observed sum within all of the permuted sums.

Calculate $Z_j = \Phi^{-1}\left(\frac{R_0 - 0.5}{Nperms}\right)$.

6) If $\min(n_{1j}, n_{2j}) \geq 2$ and $Nperms > 1000$ then

i) Generate 1,000 random permutations of sizes n_{1j} and n_{2j} from the combined sample XY_j.

ii) For each permuted sample, calculate the sum of the sample of size n_{1j}.

iii) Let R_0 equal the rank of the observed sum within the 1000 permuted sums and calculate $Z_j = \Phi^{-1}\left(\frac{R_0 - 0.5}{1001}\right)$.

STEP 3: Truncate Z-statistic for each cell

For each cell, $Z_j^* = \begin{cases} Z_j & L = 1 \\ \min(0, Z_j) & \text{otherwise} \end{cases}$.

Note that there is no truncation step if there is only one cell in the submeasure calculation.

STEP 4: Calculate the theoretical mean and variance of the truncated statistic under parity.

1. If for cell *j*, $W_j = 0$, set $ExpectedMean_j^{parity}$, $ExpectedVariance_j^{parity}$, and $ExpectedSkew_j^{parity}$ all equal to 0.
2. If $\min(n_{1j}, n_{2j}) > 6$ and $s_{1j}^2 > 0$
   a. $ExpectedMean_j^{parity} = -\frac{1}{\sqrt{2\pi}}$.
   b. $ExpectedVariance_j^{parity} = \frac{1}{2} - \frac{1}{2\pi}$
   c. $ExpectedSkew_j^{parity} = -\left(\frac{1}{2\sqrt{2\pi}} + \frac{2}{(2\pi)^{\frac{3}{2}}}\right)$
3. If $\min(n_{1j}, n_{2j}) \leq 6$ OR $s_{1j}^2 = 0$
   a. Let $N_j = \min(Nperms, 1000)$
   b. For $i = 1, \ldots, N_j$; $z_{ji} = \min\left\{0, \Phi^{-1}\left(\frac{i - 0.5}{N_j}\right)\right\}$.
   c. $\Theta_{ji} = \frac{1}{N_j}$
   d. $ExpectedMean_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}$

e. $$ExpectedVariance_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}^2 - (ExpectedMean_j^{parity})^2$$

f. $$ExpectedSkew_j^{parity} = \sum_i \Theta_{ji} z_{ji}^3 - 3ExpectedMean_j^{parity} \times ExpectedVariance_j^{parity} - \left[ExpectedMean_j^{parity}\right]^3$$

STEP 5: Calculate the initial aggregate test statistic.

$$Z_0^T = \begin{cases} Z_1 & L = 1 \\ Z^T = \dfrac{\sum_j W_j (Z_j^* - ExpectedMean_j^{parity})}{\sqrt{\sum_j W_j^2 \times ExpectedVariance_j^{parity}}} & otherwise \end{cases}$$

STEP 6: Calculate the final aggregate test statistic.

1. If L = 1, we use the cell modified Z statistic. $Z^T = Z_0^T = Z_1$.
2. If L > 1, do the following.
   a. Calculate the aggregate skewness coefficient.
   $$g_{agg} = \frac{\sum_j W_j^3 \times ExpectedSkew_j^{parity}}{6 \times \left(\sum_j W_j^2 \times ExpectedVariance_j^{parity} \right)^{\frac{3}{2}}}$$
   b. If $Z_0^T > -\frac{1 + 4g_{agg}^2}{4g_{agg}}$ or $-10^{-6} < g_{agg} < 0$ then $Z^T = Z_0^T$.
   c. Otherwise
   $$Z^T = \frac{-1 + \sqrt{1 + 4g_{agg}^2 + 4g_{agg} Z_0^T}}{2g_{agg}}$$

Proportion Performance Measures[3]

Variable definitions:

a_{1j} = Number of ILEC cases possessing an attribute of interest in cell j.

a_{2j} = Number of carrier-customer cases possessing an attribute of interest in cell j.

a_j = Number of cases possessing an attribute of interest in cell j.

NOTE: All measurements made using the number of *misses* (or negative measurement value).

STEP 1: Calculate Cell Weights.

$$W_j = \sqrt{\frac{n_{1j}n_{2j}}{n_j}\frac{a_j}{n_j}\left(1-\frac{a_j}{n_j}\right)}$$

For each cell, multiply the ILEC sample size and the carrier-customer sample size, the proportion of affected transactions and the proportion of non-affected transactions, divide by the total number of transactions, and take a square root.

STEP 2: Calculate a Z-statistic for each cell.

If $W_j = 0$ then set $Z_j = 0$.

Else, calculate the Z-statistic as $Z_j = \dfrac{n_j a_{1j} - n_{1j} a_j}{\sqrt{\dfrac{n_{1j}n_{2j}a_j(n_j - a_j)}{n_j - 1}}}$

STEP 3: Truncate Z-statistic for each cell.

For each cell, $Z_j^* = \begin{cases} Z_j & L = 1 \\ \min(0, Z_j) & \text{otherwise} \end{cases}$

Note that there is no truncation step if there is only one cell in the submeasure calculation.

STEP 4: Calculate the theoretical mean and variance of the truncated statistic under parity.

1. If for cell *j*, $W_j = 0$, set $ExpectedMean_j^{parity}$, $ExpectedVariance_j^{parity}$, and $ExpectedSkew_j^{parity}$ all equal to 0.

[3] Only perform STEP 4 if L > 1 (e.g., if this is a cell-level comparison, and there is more than one cell with CLEC activity, then perform STEP 4).

2. If $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} > 9$.
    a. $ExpectedMean_j^{parity} = -\frac{1}{\sqrt{2\pi}}$.
    b. $ExpectedVariance_j^{parity} = \frac{1}{2} - \frac{1}{2\pi}$.
    c. $ExpectedSkew_j^{parity} = -\left(\frac{1}{2\sqrt{2\pi}} + \frac{2}{(2\pi)^{\frac{3}{2}}}\right)$

3. Else, if $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} \le 9$.
    a. Let $i = \max(0, a_j - n_{2j}), \ldots, \min(a_j, n_{1j})$.
    b. Calculate $z_{ji} = \min\left\{0, \frac{n_j i - n_{1j} a_j}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}\right\}$ for each value of *i*.
    c. For each value of *i*, calculate $\Theta_{ji} = HG(i, n_{1j}, n_{2j}, a_j)$.
    d. $ExpectedMean_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}$.
    e. $ExpectedVariance_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}^2 - (ExpectedMean_j^{parity})^2$.
    f. $ExpectedSkew_j^{parity} =$
    $$\sum_i \Theta_{ji} z_{ji}^3 - 3 ExpectedMean_j^{parity} \times ExpectedVariance_j^{parity} - \left[ExpectedMean_j^{parity}\right]^3$$

STEP 5: Calculate the initial aggregate test statistic.

1. If L = 1 and $\min\left\{\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\}\right\} \le 9$,

    $Z_0^T = \Phi^{-1}\ (\alpha)$

    where $\alpha = CHG(a_{1j}, n_{1j}, n_{2j}, a_j)$.

2. If L > 1 or $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} > 9$,

$$Z_0^T = \begin{cases} Z_1 & L = 1 \\ Z^T = \dfrac{\sum_j W_j (Z_j^* - ExpectedMean_j^{parity})}{\sqrt{\sum_j W_j^2 \times ExpectedVariance_j^{parity}}} & otherwise \end{cases}$$

STEP 6: Calculate the final aggregate test statistic.

1. If L = 1, we use the cell modified Z statistic. $Z^T = Z_0^T$.
2. If L > 1, do the following.
   a. Calculate the aggregate skewness coefficient.

$$g_{agg} = \frac{\sum_j W_j^3 \times ExpectedSkew_j^{parity}}{6 \times \left(\sum_j W_j^2 \times ExpectedVariance_j^{parity} \right)^{\frac{3}{2}}}$$

   b. If $Z_0^T > -\frac{1+4g_{agg}^2}{4g_{agg}}$ or $-10^{-6} < g_{agg} < 0$ then $Z^T = Z_0^T$.

   c. Otherwise

$$Z^T = \frac{-1 + \sqrt{1 + 4g_{agg}^2 + 4g_{agg} Z_0^T}}{2g_{agg}}$$

Rate Performance Measures[4]

<u>Variable definitions:</u>

b_{1j}	=	Number of ILEC base elements in cell j.
b_{2j}	=	Number of carrier-customer base elements in cell j.
b_j	=	Total number of base elements cell j.
$r_{1j} = n_{1j} / b_{1j}$	=	ILEC sample rate of cell j.
$r_{2j} = n_{2j} / b_{2}$	=	Carrier-customer sample rate of call j.
$q_j = b_{1j} / b_j$	=	Relative proportion of ILEC elements for cell j.

STEP 1: Calculate Cell Weights.

$$W_j = \sqrt{\frac{b_{1j} b_{2j}}{b_j} \frac{n_j}{b_j}}$$

For each cell, multiply the number of ILEC base elements, the number of carrier-customer base elements and the number of transactions, divide by the total number of base elements squared, and take a square root.

STEP 2: Calculate a Z-statistic for each cell.

If $W_j = 0$ then set $Z_j = 0$.

Else, calculate the Z-statistic as $Z_j = \dfrac{n_{1j} - n_j q_j}{\sqrt{n_j q_j (1 - q_j)}}$

STEP 3: Truncate Z-statistic for each cell.

For each cell, $Z_j^* = \begin{cases} Z_j & L = 1 \\ \min(0, Z_j) & \text{otherwise} \end{cases}$

Note that there is no truncation step if there is only one cell in the submeasure calculation.

STEP 4: Calculate the theoretical mean and variance of the truncated statistic under parity.

[4] Only perform STEP 4 if L > 1 (e.g., if this is a cell-level comparison, and there is more than one cell with CLEC activity, then perform STEP 4).

1. If for cell *j*, $W_j = 0$, set $ExpectedMean_j^{parity}$, $ExpectedVariance_j^{parity}$, and $ExpectedSkew_j^{parity}$ all equal to 0.

2. If $\min(n_{1j}, n_{2j}) > 15$ *and* $n_j q_j (1 - q_j) > 9$
   a. $ExpectedMean_j^{parity} = -\dfrac{1}{\sqrt{2\pi}}$.
   b. $ExpectedVariance_j^{parity} = \dfrac{1}{2} - \dfrac{1}{2\pi}$
   c. $ExpectedSkew_j^{parity} = -\left(\dfrac{1}{2\sqrt{2\pi}} + \dfrac{2}{(2\pi)^{\frac{3}{2}}} \right)$

3. If $\min(n_{1j}, n_{2j}) \le 15$ *or* $n_j q_j (1 - q_j) \le 9$
   a. Let $i = 0, \ldots, n_j$.
   b. Calculate $z_{ji} = \min\left\{ 0, \dfrac{i - n_j q_j}{\sqrt{n_j q_j (1 - q_j)}} \right\}$ for each value of *i*.
   c. For each value of *i*, calculate $\Theta_{ji} = BN(i, n_j, q_j)$.
   d. $ExpectedMean_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}$.
   e. $ExpectedVariance_j^{parity} = \sum_{i=1}^{N_j} \Theta_{ji} z_{ji}^2 - (ExpectedMean_j^{parity})^2$.
   f. $ExpectedSkew_j^{parity} =$
   $$\sum_{\mathrm{i}} \Theta_{\mathrm{ji}} \mathrm{z}_{\mathrm{ji}}^3 - 3 ExpectedMean_j^{parity} \times ExpectedVariance_j^{parity} - \left[ExpectedMean_j^{parity} \right]^3$$

STEP 5: Calculate the initial aggregate test statistic.

1. If L = 1 and (or $n_j q_j (1 - q_j) \le 9$),
   $Z_0^T = \Phi^{-1}(\alpha)$

   where $\alpha = CBN(n_{1j}, n_j, q_j)$.

2. If L > 1 or $\min(n_{1j}, n_{2j}) > 15$ *or* $n_j q_j (1 - q_j) > 9$,

$$Z_0^T = \begin{cases} Z_1 & L = 1 \\ Z^T = \dfrac{\sum_j W_j (Z_j^* - ExpectedMean_j^{parity})}{\sqrt{\sum_j W_j^2 \times ExpectedVariance_j^{parity}}} & otherwise \end{cases}$$

STEP 6: Calculate the final aggregate test statistic.

1. If L = 1, we use the cell modified Z statistic. $Z^T = Z_0^T$.

2. If L > 1, do the following.
   a. Calculate the aggregate skewness coefficient.

$$g_{agg} = \frac{\sum_j W_j^3 \times ExpectedSkew_j^{parity}}{6 \times \left(\sum_j W_j^2 \times ExpectedVariance_j^{parity} \right)^{\frac{3}{2}}}$$

   b. If $Z_0^T > -\dfrac{1 + 4g_{agg}^2}{4g_{agg}}$ or $-10^{-6} < g_{agg} < 0$ then $Z^T = Z_0^T$.

   c. Otherwise

$$Z^T = \frac{-1 + \sqrt{1 + 4g_{agg}^2 + 4g_{agg} Z_0^T}}{2g_{agg}}$$